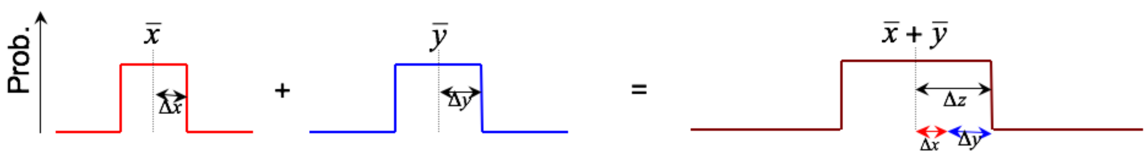
Rules for the Propagation of Uncertainty

Function	Rectangular Distribution	Normal (Gaussian) Distribution
z = x + a *where* a = constant	Δz = Δx	Δz = Δx
z = b*x *where* b = constant	Δz = |b| * Δx	Δz = |b| * Δx
z = b*x^2 *where* b = constant	Δz / z = 2 * Δx / x	Δz / z = 2 * Δx / x
z = b*x^n *where* b = constant	Δz / z = |n| * Δx / x	Δz / z = |n| * Δx / x
z = x + y	Δz = Δx + Δy	Δz = √($Δx^2$ + $Δy^2$)
z = x - y	Δz = Δx + Δy	Δz = √($Δx^2$ + $Δy^2$)
z = x * y	Δz/z = Δx/x + Δy/y	Δz/z = √($(Δx/x)^2$ + $(Δy/y)^2$)
z = x / y	Δz/z = Δx/x + Δy/y	Δz/z = √($(Δx/x)^2$ + $(Δy/y)^2$)

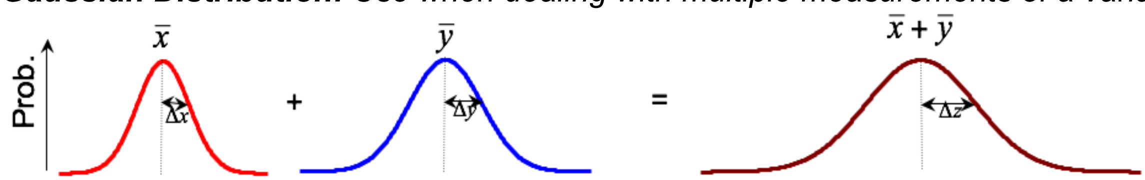
Where

Δ = uncertainty

Rectangular Distribution*: Use when dealing with precision of measuring device*



Gaussian Distribution: *Use when dealing with multiple measurements of a variable*



Examples of Dealing with ***Compound Functions*** (Combined Forms of the Functions Illustrated Above)

Example 1: Calculate the uncertainty of the acceleration caused by Earth's gravity (g) using 30 measurements of the time for a tennis ball to fall y_f meters.

Step 1: Using the equation $y_f = y_0 + v_{yo}t_f + 0.5gt_f^2$ where the positive y-axis is pointed vertically downward, the acceleration caused by Earth's gravity would equal:

$$g = 2y_f/t_f^2 = 2y_f t_f^{-2} \quad \textit{where } y_0 = 0 \text{ m and } v_{y0} = 0 \text{ m/s}$$

Step 2: Since there are 30 measurements of t_f, t_f squared in the formula, one should consider using the normal distribution uncertainty propagation rules above. This will only apply for when the percent uncertainty of y_f << percent uncertainty of t_f.

Step 3: The equation is a composite of two functions described above:

$$z = b*x^n \quad \textit{where } b = \text{constant} \qquad \text{and} \qquad z = x * y$$

Step 4: Let $h = 2t_f^{-2}$ so the uncertainty of h will be:

$$\Delta h / h = |-2| * \Delta t_f / t_f = 2\Delta t_f / t_f$$

Step 5: Rewrite the equation for g to include the new variable, h:

$$g = y_f * h$$

Step 6: The uncertainty of g will now be:

$$\Delta g/g = \sqrt{((\Delta y_f/y_f)^2 + (\Delta h/h)^2)}$$

Step 7: Substitute the $\Delta h / h$ calculated in Step 4:

$$\Delta g/g = \sqrt{((\Delta y_f/y_f)^2 + (2\Delta t_f / t_f)^2)} = \sqrt{((\Delta y_f/y_f)^2 + 4(\Delta t_f / t_f)^2)}$$

Example 2: Calculating the uncertainty of velocity using the position of a falling ball in successive frames in a movie.

Step 1: Use the equation $v = (y_i - y_{i-1}) / (t_i - t_{i-1})$ to calculate v.

Step 2: Both the uncertainty of y and t are based on the rectangular distribution of uncertainty.

Step 3: The equation is a composite of two functions described above:

$$z = x - y \qquad \text{and} \qquad z = x / y$$

Step 4: Let $h_1 = y_i - y_{i-1}$ so the uncertainty of h will be:

$$\Delta h_1 = \Delta y + \Delta y = 2\Delta y \qquad \text{where } \Delta y_i = \Delta y_{i-1} = \Delta y$$

Step 5: Let $h_2 = t_i - t_{i-1}$ so the uncertainty of h will be:

$$\Delta h_1 = \Delta t + \Delta t = 2\Delta t \qquad \text{where } \Delta t_i = \Delta t_{i-1} = \Delta t$$

Step 6: Rewrite the equation for v to include the new variables, h_1 and h_2:

$$v = h_1 / h_2$$

Step 7: The uncertainty of v will now be:

$$\Delta v/v = \Delta h_1/h_1 + \Delta h_2/h_2$$

Step 8: Substitute the h_1, Δh_1, h_2, and Δh_2 defined in Steps 4 & 5:

$$\Delta v/v = 2\Delta y / (y_i - y_{i-1}) + 2\Delta t / (t_i - t_{i-1}) = 2\,(\Delta y / (y_i - y_{i-1}) + \Delta t / (t_i - t_{i-1}))$$