Using Declinations Circles

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For all practical purposes for life on Earth, the Sun is located in the center of our solar system (it does move due to the pull of the planets as they orbit the Sun), yet to objects on the Earth's surface, the Sun appears to move in a *circular* path across the sky (Figure 1). It takes 24 hours to complete the circle, and because the Earth rotates from west to east (Figure 2) the sun rises in the eastern sky and sets in the western sky. How it appears to move depends on the day of year, latitude, and Earth's tilt (the angle between the rotational and orbital axes called *obliquity* and illustrated in Figure 2).

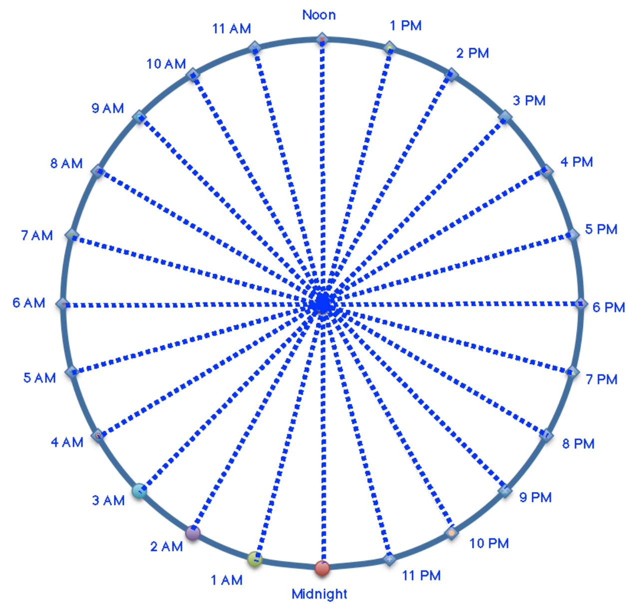


Figure 1. Hourly positions of the sun's apparent motion for 24 hours. The Sun moves 15° every hour (360°/24 hours). The sun is highest in the sky at local noon, lowest in the sky/furthest below the horizon at local midnight.

After detailed and patient observation, humans have used these apparent motions as a *compass*, *clock*, and *calendar* for thousands of years. And there many more uses too: designing houses to be energy efficient, locating windows on buildings, designing gardens, and understanding Earth's seasons and climate regimes.

There are four key dates to recognize (Figure 2 and Table 1). A *solstice* is when the Sun is directly overhead a person standing on one of the tropic circle latitudes, 23.5ºN or S, at local noon. An *equinox* occurs when the Sun is directly overhead a person standing on the Equator at local noon. Local noon is the time of day when the Sun is highest in the sky, and it also occurs when the sun is crossing your longitude.

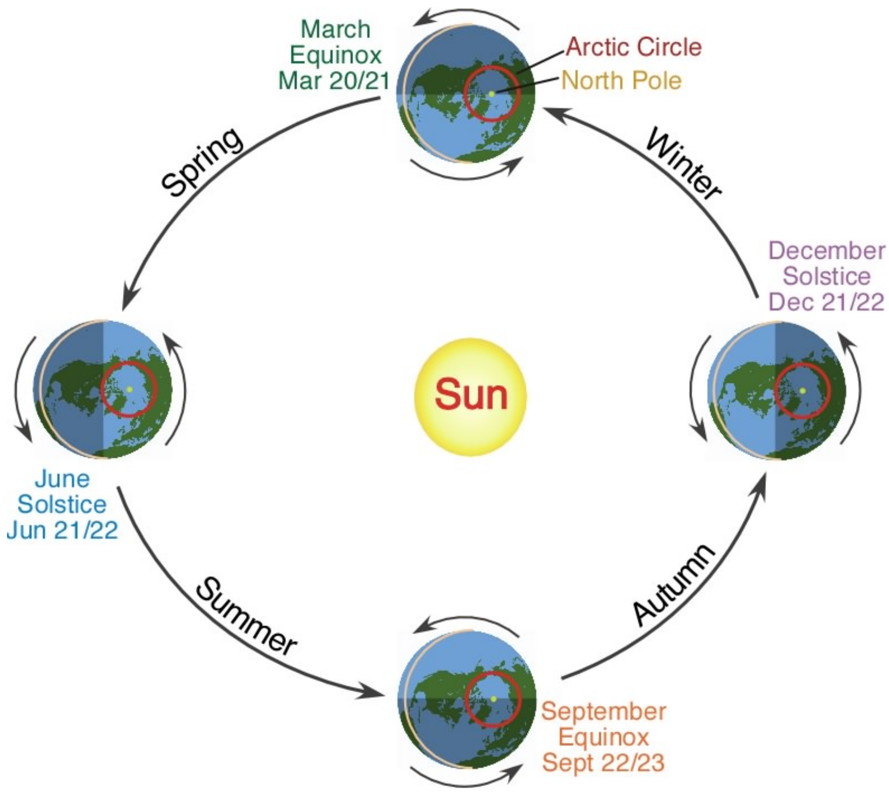


Figure 2. The Earth rotates from west to east as it orbits the Sun. The orbit is slightly elliptical, and the Sun is at one of the two foci of our elliptical orbit. The solstices occur when the Earth tilt is pointed toward/ away from the Sun, and the equinoxes when the tilt is not pointed toward the Sun. In this diagram, the labeled seasons refer to the Northern Hemisphere; they are opposite for the Southern Hemisphere (see Table 1). Image from http://www.physicalgeography.net/fundamentals/images/seasons.jpg

Date	Sun's Declination	Northern Hemisphere Season	Southern Hemisphere Season
June 21	23.5ºN	Beginning of summer	Beginning of winter
September 21	0º	Beginning of fall	Beginning of spring
December 21	23.5ºS	Beginning of winter	Beginning of summer
March 21	0º	Beginning of spring	Beginning of fall

Table 1. Note these are general dates (+/- 2 days) since the Earth orbits the Sun in 365.25 days. The declination is illustrated in Figure 3.

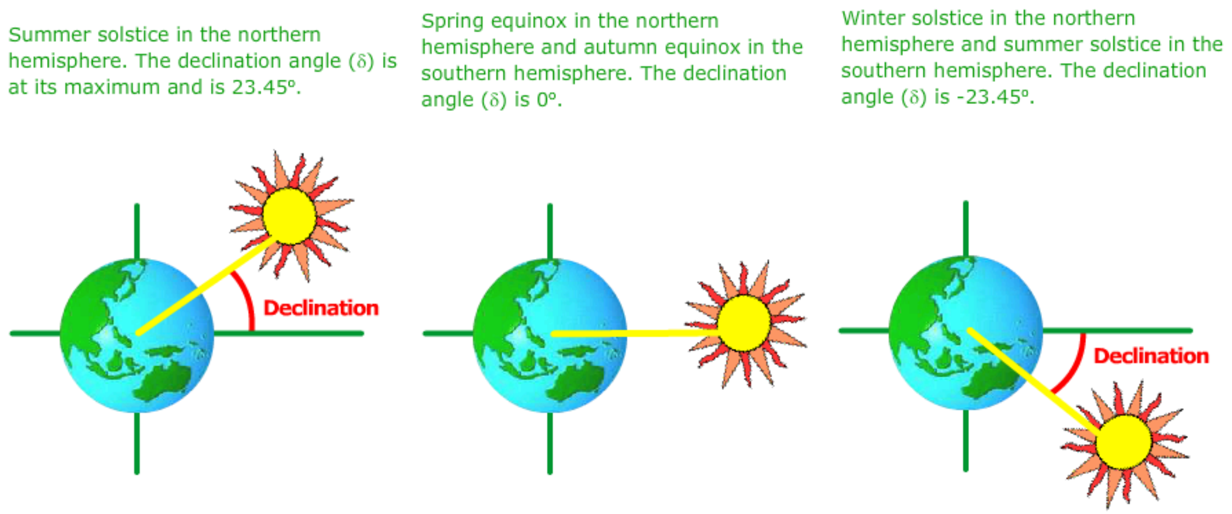


Figure 3. Despite the fact that the Earth revolves around the Sun, it is simpler to think of the Sun revolving around a stationary Earth. This requires a coordinate transformation. Under this alternative coordinate system, the Sun moves around the Earth. (From http://pveducation.org/pvcdrom/properties-of-Sunlight/declination-angle).

The diagrams in Figure 4 were generated for 42ºN (roughly the latitude of Boston, MA) during the June 21 solstice at *local noon* (the Sun is highest in the sky) using the website http://astro.unl.edu/naap/motion3/animations/Sunmotions.html. The yellow circle represents the ***declination circle***, the path of the Sun during a given day of the year for a specific location on Earth.

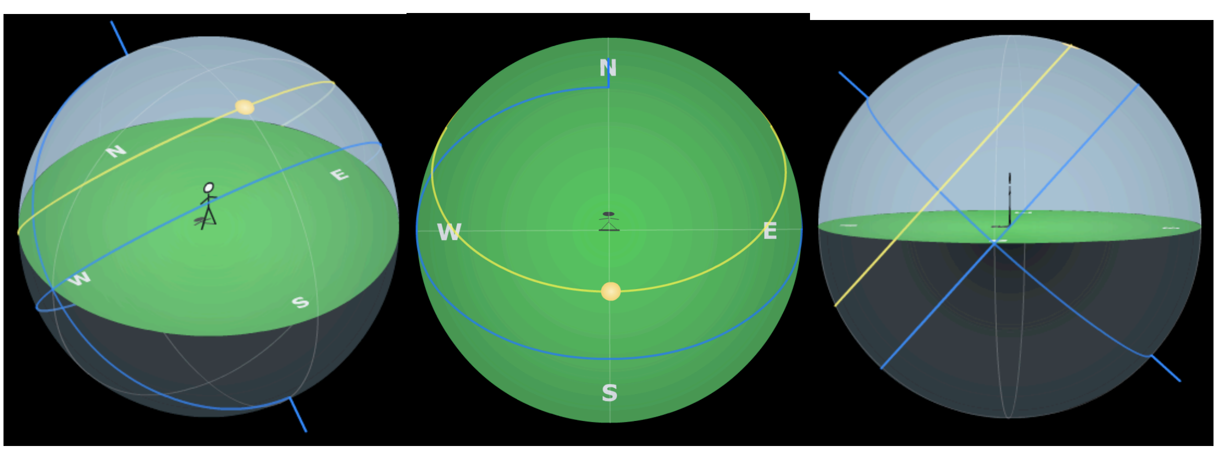


Figure 4. Various views of a declination circle for 42ºN on June 21. These are very useful for seeing when and where the Sun rises and sets, where it was located at local noon and midnight, and how much day light occurred (amount of the yellow line is above the horizon). Since it is easier to draw straight lines than circles, we will use the diagram on the right to make our own declination circles.

We are going to use a template to “flatten” the 3D view of declination circles to make it easier to draw and study declination circles (Figure 5). This circle is oriented vertically, so the horizontal green line represents the *horizon* (what a person would see standing in a large, flat field), and the vertical gray line is the *zenith* (representing the point directly overhead a person). North is typically on the left part of the horizon, south on the right, which means looking into the paper at the intersection of the horizon and zenith is east, and away from, or out of, the paper, west.

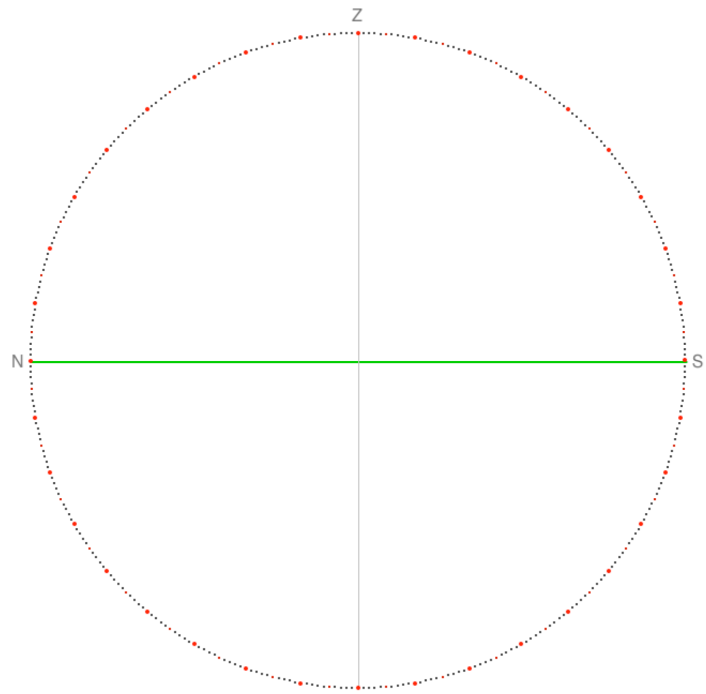


Figure 5. Template to create declination circles. You may create and save templates using the Sun-Earth Connection software at sciencepickle.com.

How to Create Declination Circles

The two key pieces of information you need are your *latitude* and the Earth's *obliquity* (or tilt), which for our purposes, is 23.5º. The obliquity is the angle of the Earth’s axis of rotation relative to its orbital axis (which is perpendicular to the plane of orbit). Figure 6 illustrates how Earth would be tilted relative to its orbit around the Sun with obliquities of 53º and 17º.

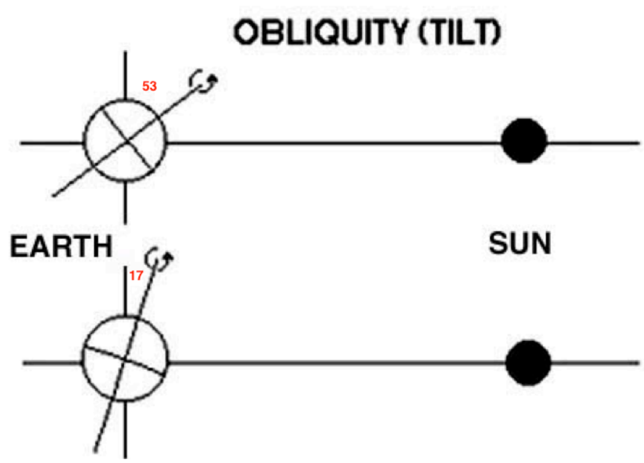


Figure 6. How the Earth's axis of rotation would change for obliquities of 53º and 17º.

Step 1: **Draw a declination circle for an *equinox*** since these are the easiest declination circles to draw for any location because the line goes through the center of the circle. In this example, use ***60ºN*** (Figure 7). The declination circle slopes toward the north for this latitude, and the steepness is inversely related to the latitude: the declination circle is oriented the degrees latitude from zenith (which is why it is helpful to label the zenith line).

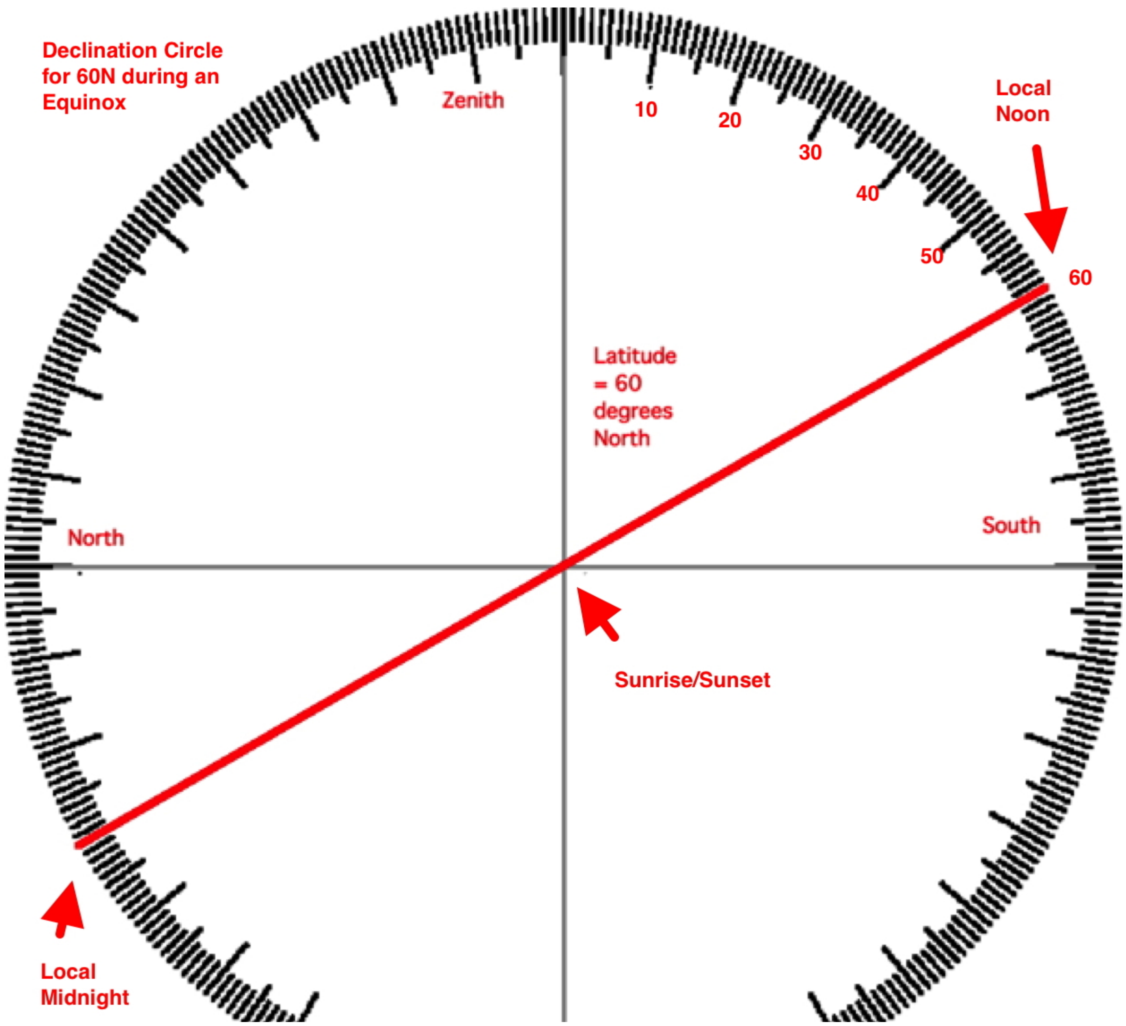


Figure 7. Declination circle for 60ºN during an equinox.

Why draw the equinox first? During this day, all declination circles for every location on Earth pass through the intersection of the horizon and zenith, which is the center of the tick-marked circle so this is the easiest line to draw. This means that for all latitudes except the Poles, the Sun rises exactly east (the intersection of the declination circle and the horizon) and sets exactly west during the equinox. At the North and South Poles, the Sun remains in the horizon for 24 hours during the equinox. Also, consider that there is only one direction you can step when standing at the North Pole – south (and vice versa for the South Pole)! There are no other directions, so the Sun could not rise exactly east at the Poles – ever.

Since the declination circle during the equinox passes through the center of the ticked-mark circle, exactly half the declination circle is above the horizon (daytime) and half below the horizon (nighttime), so the day is evenly split between day and night: 12 hours of day light, 12 hours of darkness.

Where the declination circle intersects the ticked-marked circle of the template represent two important times of the day: *local noon* (above the horizon) and *local midnight* (below the horizon). The Sun is highest in the sky at local noon (Sun is crossing the longitude of the observer), and farthest below the horizon (or lowest in the sky if the Sun doesn't set that day) at local midnight. Notice that the Sun is only 30º above the horizon at local noon (90º (or zenith) - 60º = 30º).

Additional Way to Visualize Declination Circles: Use a globe or a ball in a darkened room and a flashlight. Rotate the ball from west to east to see how a given latitude is illuminated during a revolution. If possible, use clay to anchor a straw or toothpick so you can see shadow movement on the globe. The shadow helps visualize if the Sun rises north of east, east, or south of east.

Step 2: **Draw the declination circle for the *summer* solstice.** Since 60ºN is in the Northern Hemisphere, the summer solstice occurs on June 21 (Figure 8), which is the time the Sun is located "over" the Tropic of Cancer (23.5ºN). You only need to locate two points to draw the declination circle, so go 23.5º (remember, this is the Earth's obliquity) *toward the north* for both local noon and local midnight for the declination circle of the equinoxes (drawn during Step 1).

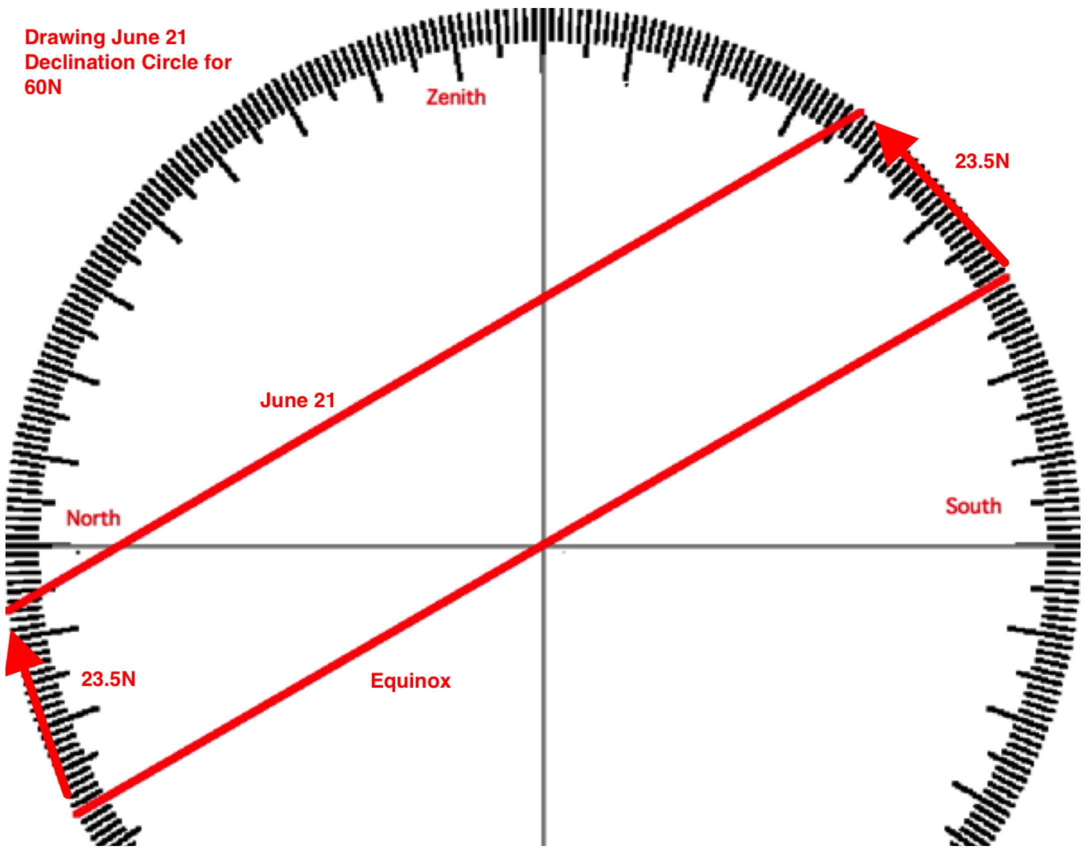


Figure 8. Declination circle for 60ºN during June 21, the Northern Hemisphere summer solstice.

If drawn correctly, the new declination circle will be *parallel* to the first one drawn for the equinoxes. Notice that the majority of the declination circle is above the horizon, so the amount of daylight is much longer than 12 hours. In a section that follows, the amount of daylight can be approximated using this diagram. Look at the intersection of the declination circle and the horizon: the Sun rises well north of east, and sets well north of west, but the Sun is well south of you at local noon. Using the diagram, the Sun is 53.5º above the horizon at local noon (30º + 23.5º = 53.5º).

Step 3: **Draw the declination circle for the *winter* solstice.** Since the location is in the Northern Hemisphere, the winter solstice will occur on December 21 (Figure 9), which is the time the Sun is located "over" the Tropic of Capricorn (23.5ºS). Go 23.5º toward the south for both local noon and local midnight for the declination circle of the equinoxes (drawn in Step 1).

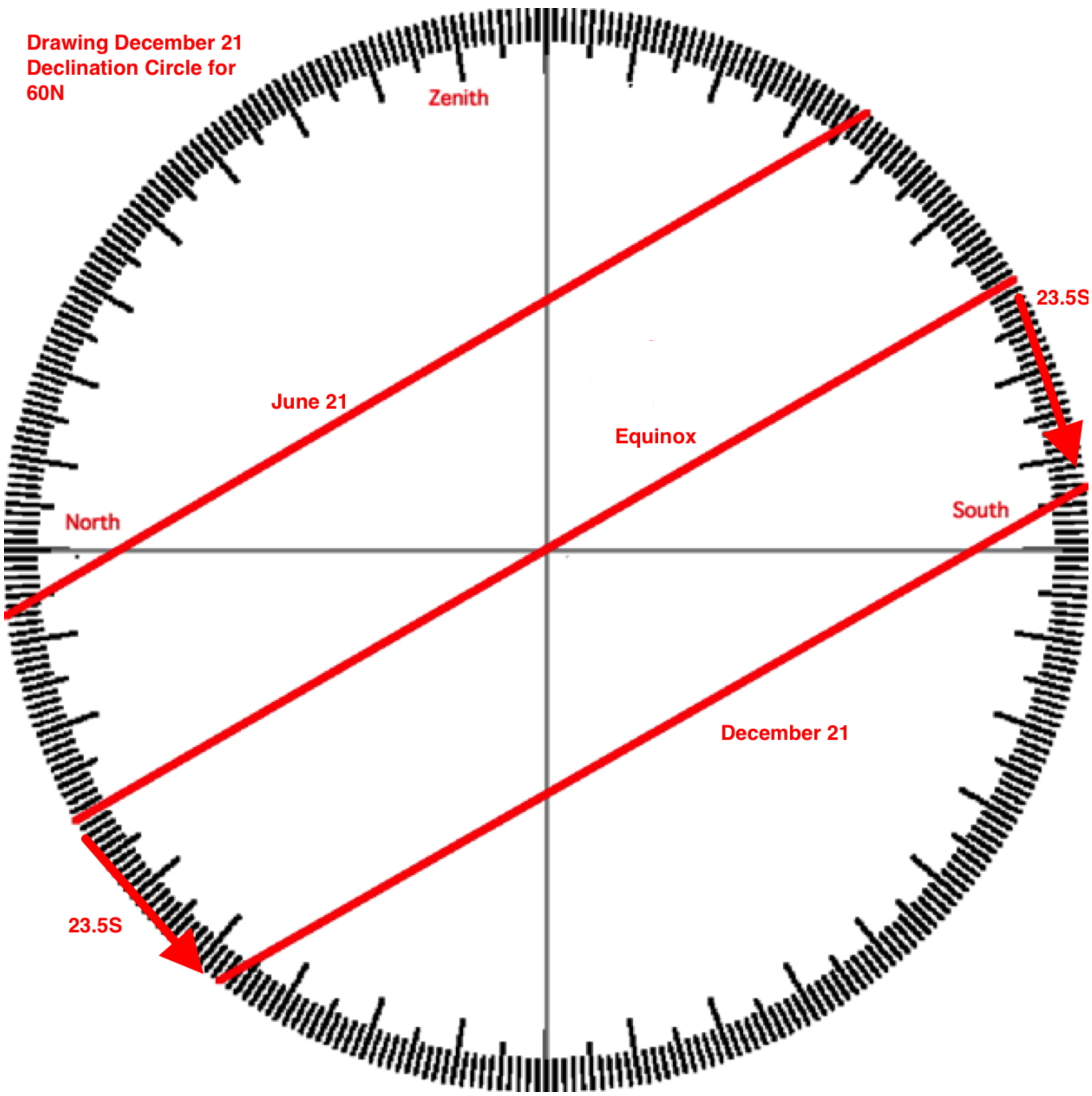


Figure 9. Declination circle for 60ºN during December 21, the Northern Hemisphere winter solstice.

Now the majority of the declination circle is below the horizon, so the amount of daylight is much shorter than 12 hours, and it is equal to the amount of darkness during the summer solstice. The Sun rose well south of east, and set well south of west (remember, look at the intersection of the declination circle and the horizon), and the Sun was still south of you at local noon - but it is 6.5º above the horizon (30º-23.5º).

Notice the symmetry between the winter and summer solstice declination circles about the equinox declination circle: the location Sun rise and Sun set for the two solstices are symmetric about east and west, respectively; the time of Sun rise an Sun set for the two solstices are symmetric about 6 AM and 6 PM, respectively:

$$\text{6 AM} - \text{Sunrise}_{\text{Summer Solistice}} = \text{Sunrise}_{\text{Winter Solstice}} - \text{6 AM}$$

$$\text{Sunset}_{\text{Summer Solistice}} - \text{6 PM} = \text{6 PM} - \text{Sunset}_{\text{Winter Solstice}}$$

Also, the amount of daylight and nighttime for the solstices are symmetric about 12 hours, meaning the amount of daylight during June 21 equals the amount of darkness during December 21, and vice versa during June 21.

Remember, declination circles represent 24 hours of apparent motion of the Sun. In this example for ***50ºN*** at an equinox (left) and the solstices (right) (Figure 10), the hourly positions of the Sun are shown. Notice for the equinoxes, the Sun rises at 6 AM due east of the tree, is located 40º above the horizon at local noon, and sets at 6 PM due west. For the summer solstice, there is much more daylight (roughly 16 hours), rises north of east, sets north of west, and is 63.5º (40º+23.5º) above the horizon at local noon. Conversely for the winter solstice, there is roughly 9 hours (24–16) sunlight, and the Sun rises south of east, sets south of west, and is located 17.5º above the horizon at local noon (40º-23.5º).

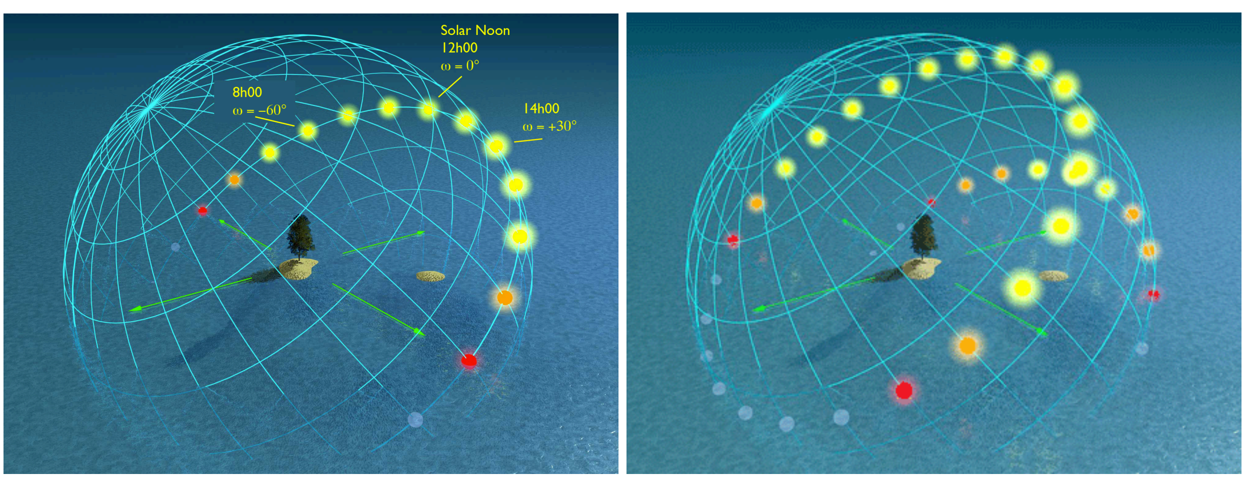


Figure 10. Hourly positions of the Sun for 50ºN during the equinox (left) and June 21 and December 21. Images from https://www.e-education.psu.edu/eme810/node/556#.

Declination Circles for Other Days of the Year

So we can draw declination circles for four days of the year, but what about the other 361 days? You may use the circles already drawn to help bracket declination circles for other times of the year. In Figure 11, declinations circles for the 21st of each month are drawn for 42ºN. Notice that the spacing between months is not even - there is a greater change during the months around the equinox compared to around the solstices. This is similar to how the Sun changes angle above the horizon most at sunrise and sunset and least at local noon.

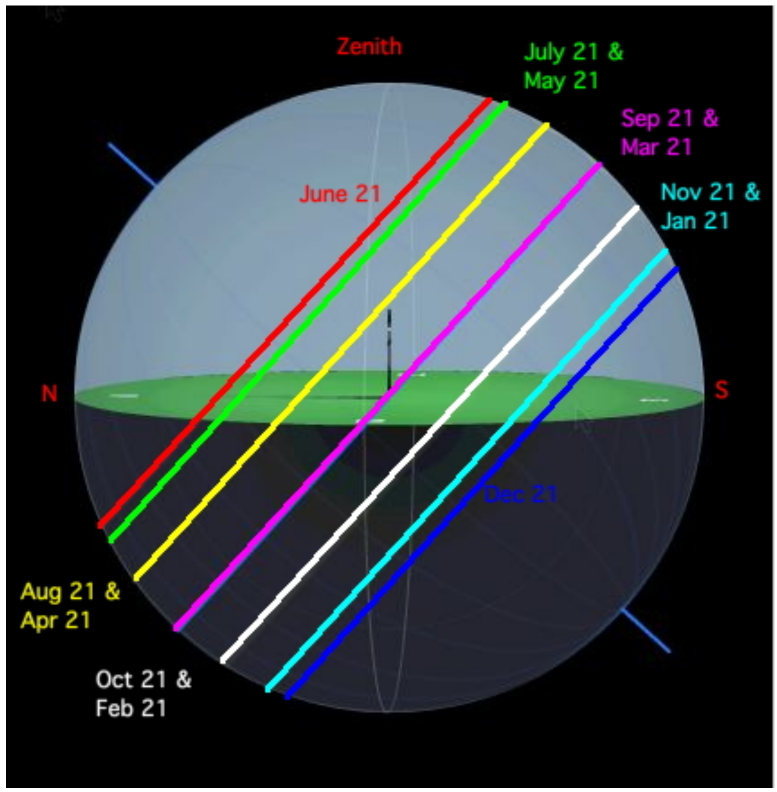


Figure 11. Declination circles for the 21st of each month for 42ºN.

To calculate the number of degrees away from the equinox declination circle, referred to as *declination or declination angle*, use the following graph (Figure 12). The declination for the summer solstice in the northern hemisphere is 23.5ºN or +23.5º; for the southern hemisphere, it is 23.5ºS or -23.5º.

Once you have drawn the declination circle for the chosen day of the year, use the information to plan your day, outing, how and when to photograph or shoot your movie, or where to plant your garden or what plants you should consider growing.

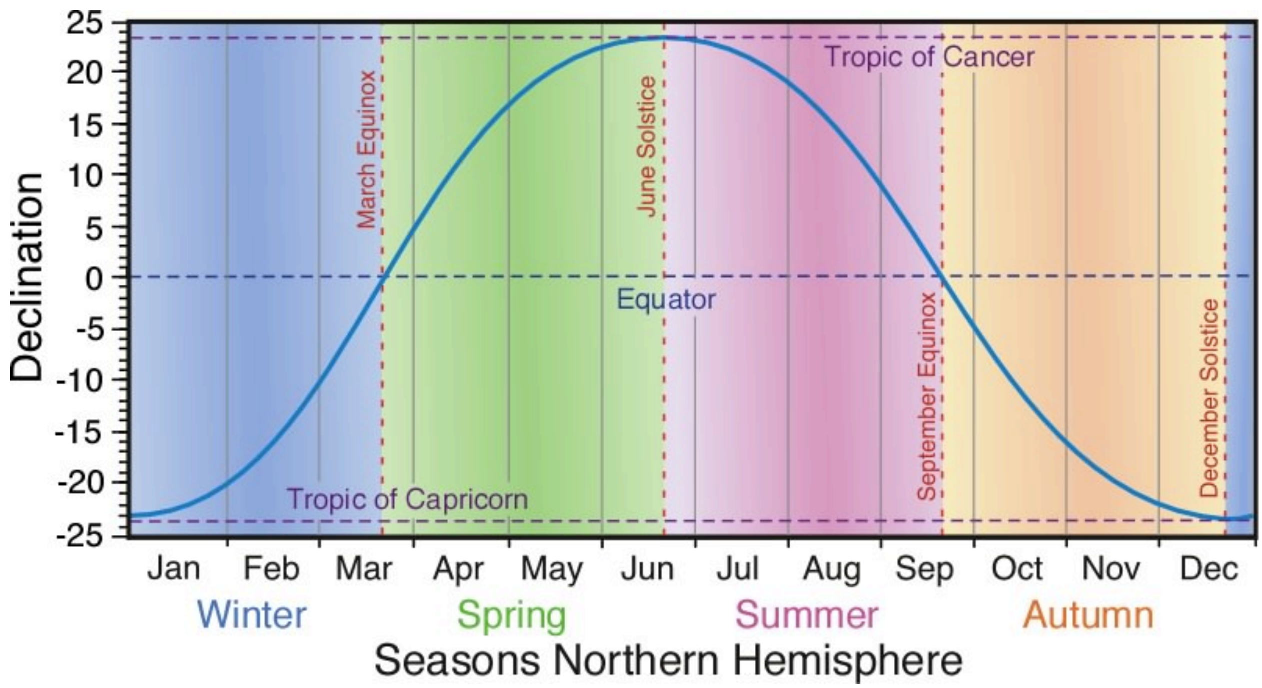


Figure 12. The Sun's declination for one year. Image from *http://www.physicalgeography.net/fundamentals/6h.html*.

If you would like to use a formula to calculate declination, δ, on any given day (from http://holbert.faculty.asu.edu/eee463/SolarCalcs.pdf):

$$\delta = 23.5^\circ * \sin(360^\circ * (284 + n) / 365)$$

Where:

δ = declination angle (degrees);

n = the day number, such that n = 1 on the 1st January.

Challenge 1: Draw the declination circle for 30ºS on August 1.

Step 1: Find the declination for August 1. Using the red lines below, the declination is 19ºN or +19º.

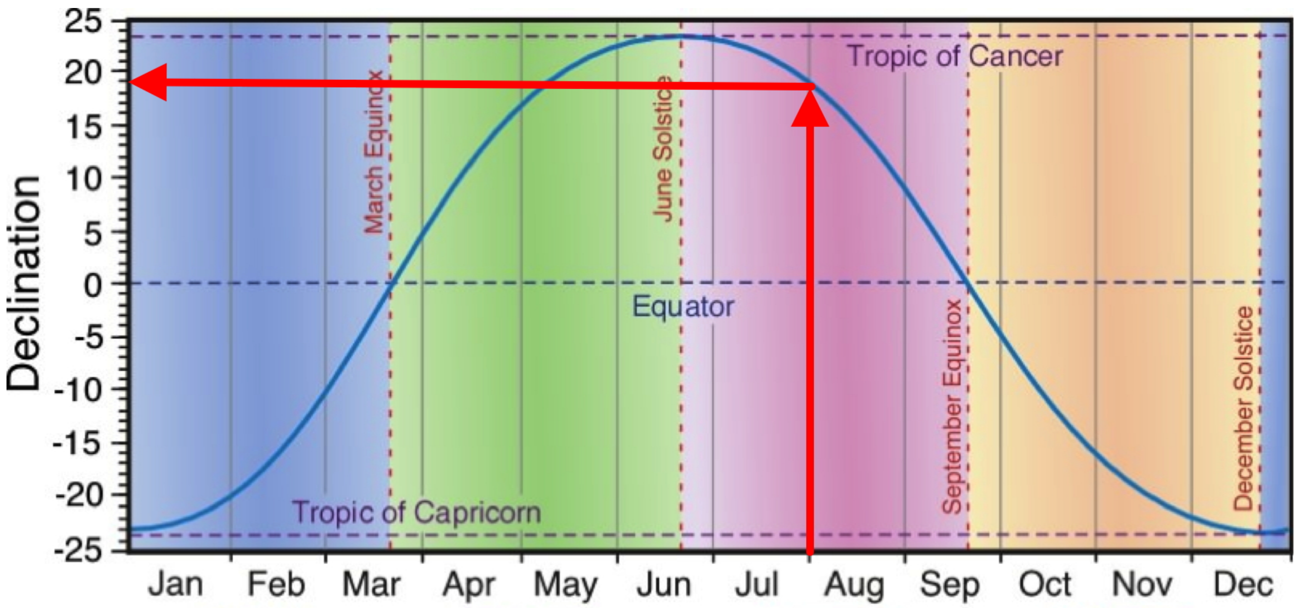


Figure 13. Calculating the declination for August 1.

Step 2: Draw the declination circle for the equinox for 30ºS (Figure 14). Remember, go 30º from the zenith, draw a line that slopes to the south (Southern Hemisphere) and goes through the center of the circle.

Step 3: Draw the declination circle for August 1 by 19º toward the north from both local noon and local midnight (Figure 14).

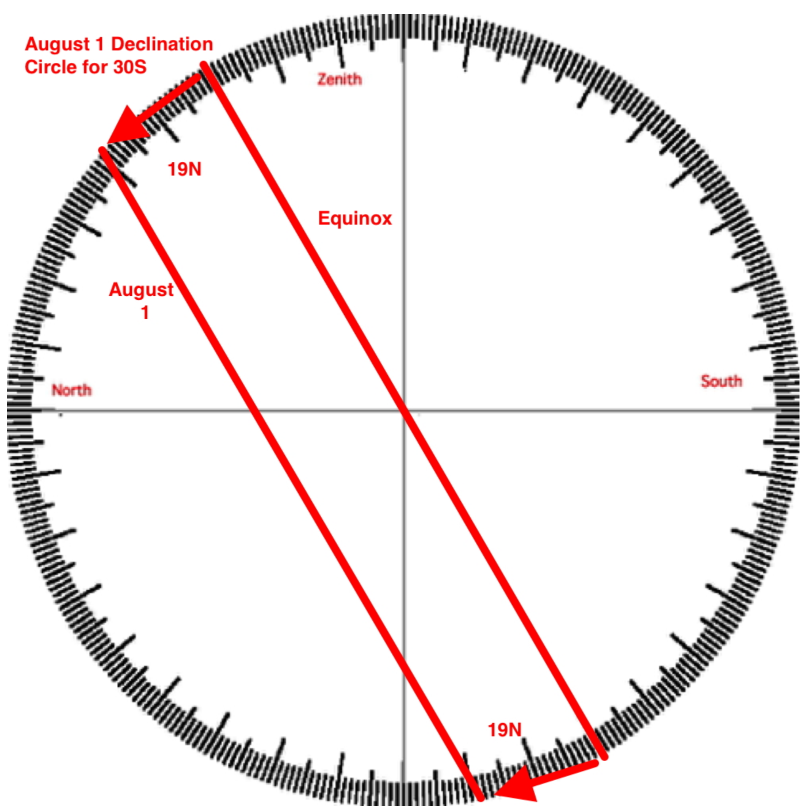


Figure 14. Declination circle for 30ºS on August 1.

Challenge 2: Determine the latitude and dates(s) if given a declination circle

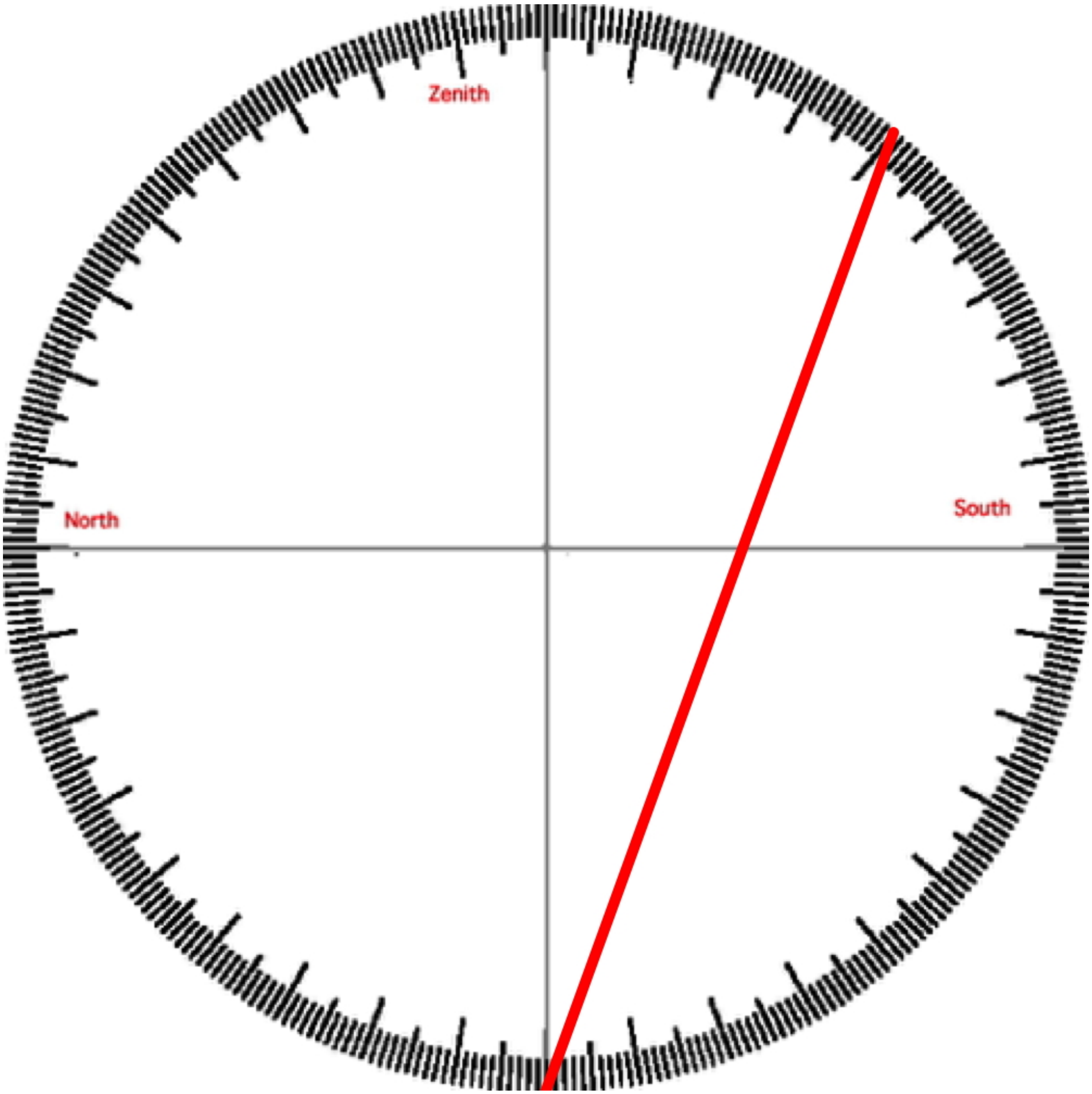


Figure 15. Use this declination circle to calculate the latitude and day(s) of the year.

Step 1: Create a declination circle that is *parallel* to the given declination circle but this one goes through the center of the circle (this will be the equinox declination circle) (Figure 16). To make sure it is parallel, go the same number of degrees from the local noon and midnight points. In this example, go 20º from both the local noon and midnight points.

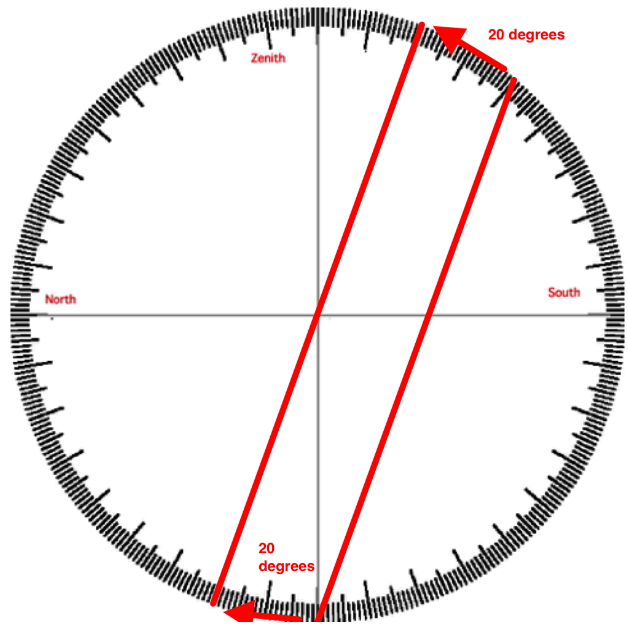


Figure 16. Draw the declination circle for the equinox that is parallel to the given declination circle.

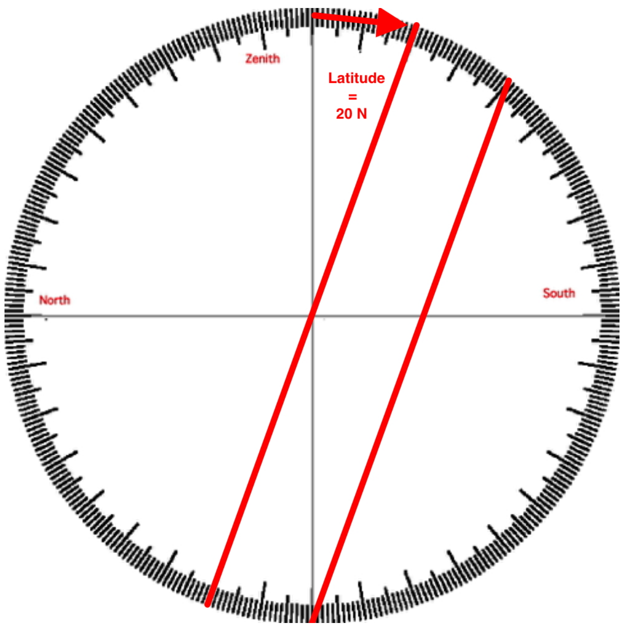
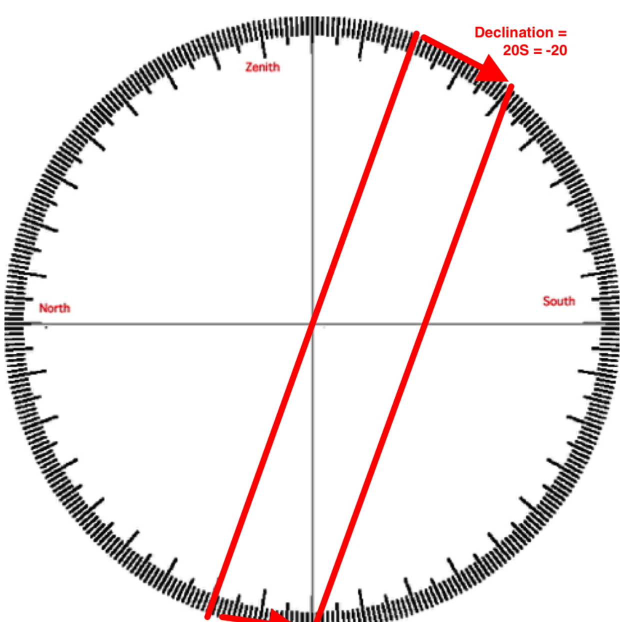


Figure 17. Calculate the latitude using the declination circle for the equinox.

Step 2: Calculate the latitude and hemisphere from the equinox declination circle by measuring the number of degrees from zenith and the slope of the circle (Figure 17). In this case, the latitude is 20ºN (the declination circle slopes northward).



Step 3: Calculate the declination (the angle between the equinox declination circle and the challenge declination circle, which is 20ºS or -20º (Figure 18).

Figure 18. Calculate declination from equinox declination circle to the given one.

Step 4: Find the dates that have a declination of -20º. There are two days with this declination: February 1 and November 14 (Figure 19).

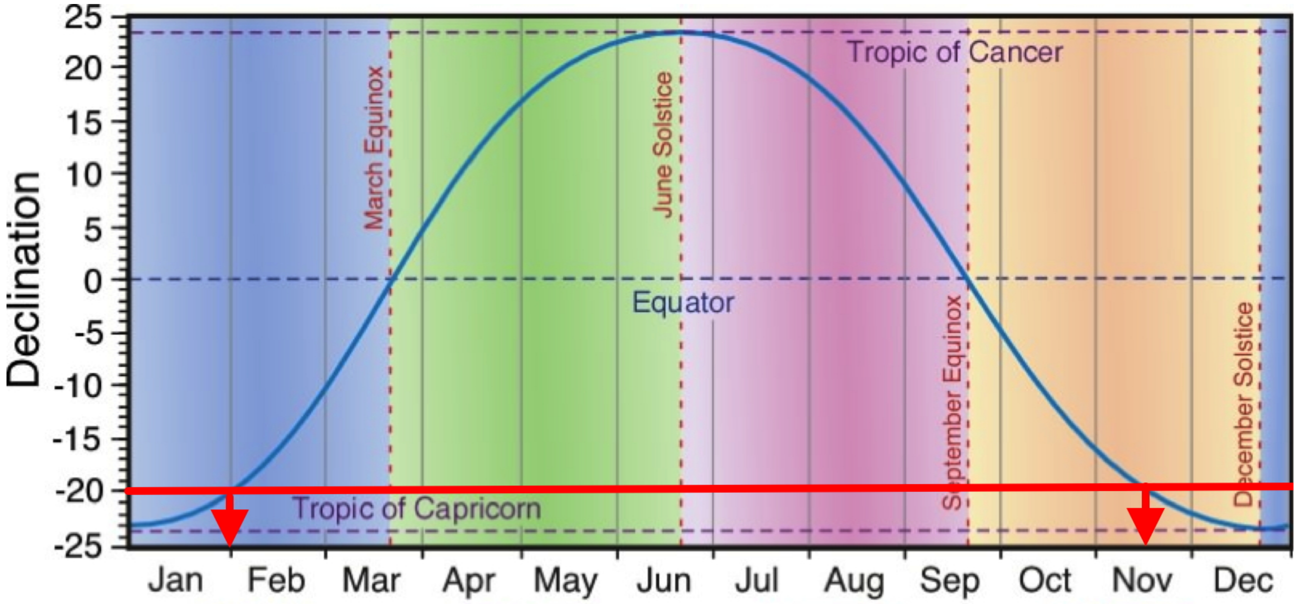


Figure 19. Days of the year with a declination of -20º.

Test yourself - do these declination circles makes sense (Figure 20)? Note: north is to the right.

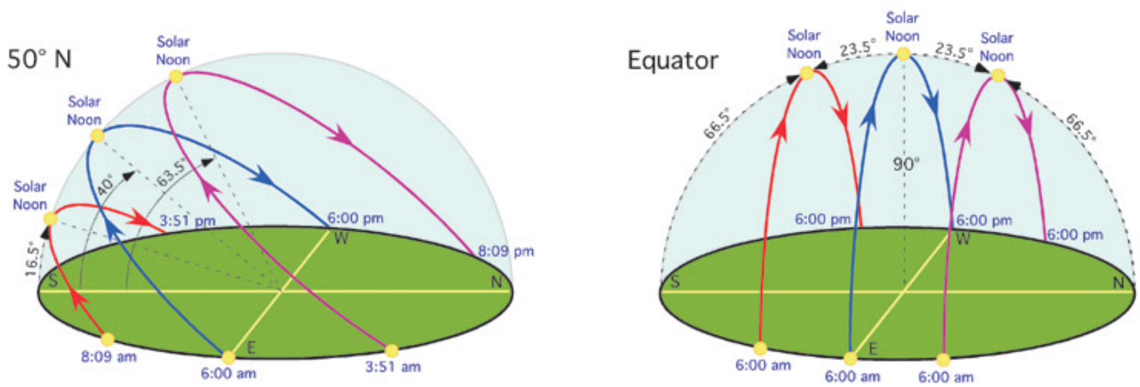


Figure 20. Declination circles for 50ºN and 0º. From http://notesfromnoosphere.blogspot.com/2012/05/simple-geometry-of-Sun-paths.html