Calculating Time with Declinations Circles

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We can do more than say that there is more daytime than nighttime – we can make quite good estimates of sunrise and sunset using the 2D declination circles. To be precise, we would need to take into account the refraction of sunlight as it travels through the Earth's atmosphere, the effect of Earth's slightly eccentric (elliptical) orbit, and the longitude (to take into the account the time zone).

Recall that the Sun moves 15° along its declination circle each hour, but there are other ways to view its motion: one is based relative to the horizon (Figure 1). Using this perspective, the Sun changes angle relative to the horizon fastest at sunrise and sunset and least during local noon.

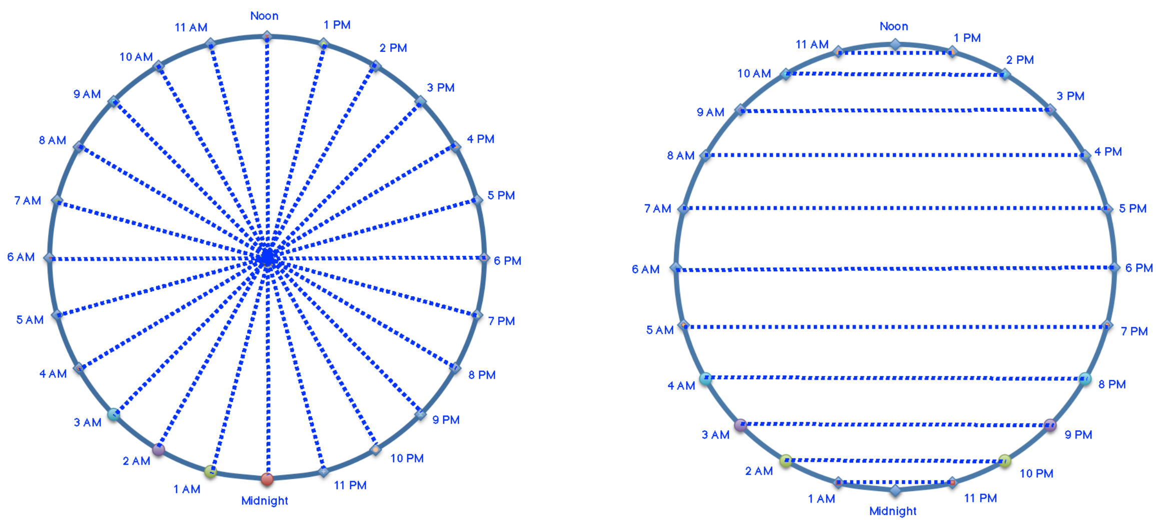


Figure 1. Two ways to view how the Sun moves in a circle: 15° each hour (left) or relative to the horizon (right).

Now consider how we viewed the declination circle in 2D – it is a straight line with the lowest point being local midnight and the highest being local noon. Halfway between would be either 6 AM or 6 PM. This is true for any declination circle except for those at the poles since sun time does not exist since the Sun moves parallel to the horizon at the Poles (Figure 2).

Figure 2. Location of Sun by time on a 2D declination circle.

To calculate the locations of other hours of the day on a declination circle, we need to use trigonometry (Figure 3). The red line represents the 2D declination circle, and its length is two times the radius of the 3D declination circle, which we will label as **r**. In this example, it is 2 PM, so the angle of the Sun from the horizon along the declination circle is 60°. The *distance from the midpoint* is *r*sin(60°) of the red line*. We can do this for all of the hourly positions of the Sun (Figure 4).

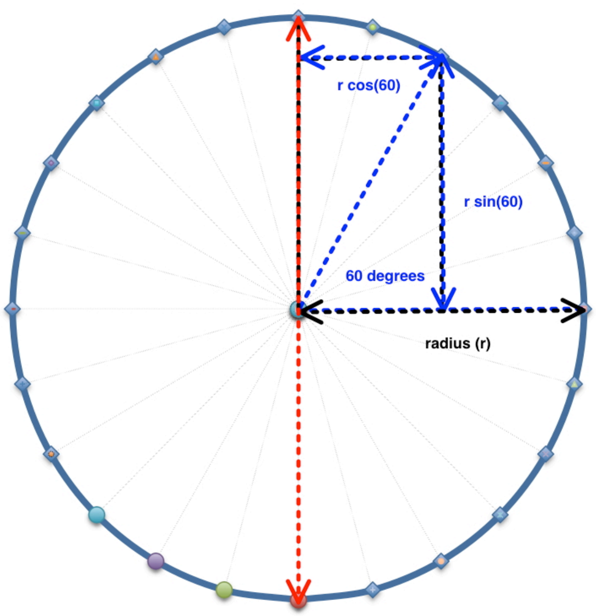


Figure 3. Calculating the position of the Sun for 2 PM on a 2D declination circle (represented by the red line).

Figure 4. Hourly Sun positions.

Use Table 1 to calculate the percent of the distance from the center of the 2D declination circle.

Time of Day	Percent Distance from Midpoint of 2D Declination Circle
Local Noon	+100% (above midpoint)
11 AM & 1 PM	97%
10 AM & 2 PM	87%
9 AM & 3 PM	71%
8 AM & 4 PM	50%
7 AM & 5 PM	26%
6 AM & 6 PM	0% (at midpoint)
5 AM & 7 PM	-26%
4 AM & 8 PM	-50%
3 AM & 9 PM	-71%
2 AM & 10 PM	-87%
1 AM & 11 PM	-97%
Local Midnight	-100% (below midpoint)

Table 1. Scale to estimate time of sunrise and sunset.

Example: Use the declination circle for 20 N on February 1 (This was Figure 15 in the Using Declination Circles document) and find the time of sunrise, sunset, length of daytime and length of nighttime (Figure 5).

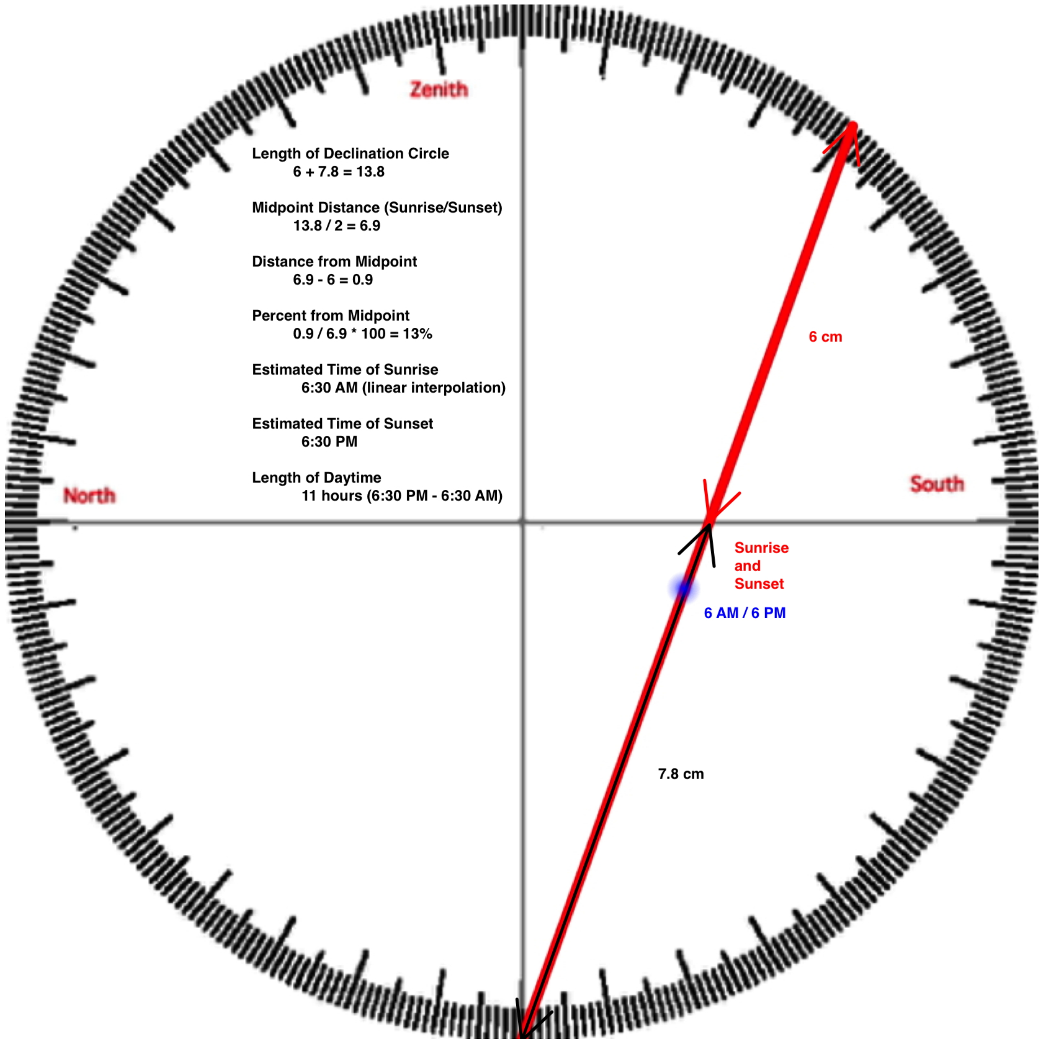
Figure 5. Calculating the time of sunrise, sunset, and duration of day light.

Step 1: Measure the length of the declination circle above and below the horizon. In this example, 6 cm was above and 7.8 cm below.

Step 2: Add together to calculate the length of the 2D declination circle (13.8 cm).

Step 3: Find the midpoint, which is the position of 6 AM/PM: 6.9 = 13.8 / 2

Step 4: Calculate the distance from the sunrise to 6 AM: 6.9 – 6 = 0.9 *above midpoint.*



Step 5: Calculate the percent distance from the midpoint: 0.9 / 6.0 x 100 = 13%

Step 6: Interpolate the time from Table 2: 13% is roughly half of 25%, so sunrise is 6:30 AM and sunset is 6:30 PM.

Step 7: Subtract sunrise from sunset to calculate the length of daytime: 11 hours = 6:30PM – 6:30 AM

Step 8: Length of nighttime = 24 – hours sunlight = 24 – 11 = 13 hours.